

Coq 常用证明

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来源网站: [链滴](#)

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化简等式

对于简单的等式，可以使用 `simpl` 将等式进行化简，然后用 `reflexivity` 即等号的自反性证明。

```

Theorem lus_O_n : forall n : nat,
  n <= 0 + n <= n <= n
Proof.
  intros n.
  simpl.
  reflexivity.
Qed

```

Coq 系统中可以自动化简，故只使用 `reflexivity` 也可证明定理。

```

Theorem lus_O_n : forall n : nat,
  n <= 0 + n <= n <= n
Proof.
  intros n.
  reflexivity.
Qed

```

根据条件重写式子

当定理中含有条件时，如下 `n=m` 为条件，则需要将条件引入，即 `intros H`，此时 `H` 表示 `n=m` 这个条件。然后使用 `rewrite` 命令，根据条件重写式子。有两种方式使用 `rewrite`：

- `rewrite H`：得到 `n+n=n+n`
- `rewrite -H`：得到 `m+m=m+m`

有时需要分析应该使用哪种方式。

```

Theorem lus_id_example : forall n m : nat,
  n <= 0 + n <= n <= m <= m
Proof.
  intros n m.
  rewrite -H.

```

```

highlight-n">n</span> <span class="highlight-o">+</span> <span class="highlight-n">n</sp
n> <span class="highlight-o">=</span> <span class="highlight-n">m</span> <span class
"highlight-o">+</span> <span class="highlight-n">m</span><span class="highlight-o">.<
span>
</span></span><span class="highlight-line"><span class="highlight-cl"><span class="high
ight-kn">Proof</span><span class="highlight-o">.</span>
</span></span><span class="highlight-line"><span class="highlight-cl"> <span class="hi
hlight-k">intros</span> <span class="highlight-n">n</span> <span class="highlight-n">
</span><span class="highlight-o">.</span>
</span></span><span class="highlight-line"><span class="highlight-cl"> <span class="hi
hlight-k">intros</span> <span class="highlight-n">H</span><span class="highlight-o">.<
span>
</span></span><span class="highlight-line"><span class="highlight-cl"> <span class="hi
hlight-k">rewrite</span> <span class="highlight-o">&lt;</span> <span class="highlight-n
">H</span><span class="highlight-o">.</span>
</span></span><span class="highlight-line"><span class="highlight-cl"> <span class="hi
hlight-kp">reflexivity</span><span class="highlight-o">.</span> <span class="highlight-k
">Qed</span><span class="highlight-o">.</span>
</span></span></code></pre>
<h3 id="分类讨论">分类讨论</h3>
<p>分类讨论是根据变量的类型进行分情况讨论。如下，引入 <code>n</code>, <code>n</cod
> 的类型为 <code>nat</code>。<code>nat</code> 有两种情况，第一种情况是 <code>O</c
ode>, 即数字 0, 第二种情况为 <code>S n'</code>, 其中 <code>n'</code> 为需要的参数。</
>
<pre><code class="language-coq highlight-chroma"><span class="highlight-line"><span cl
ss="highlight-cl"><span class="highlight-kn">Theorem</span> <span class="highlight-n">
lus_1_neq_0</span> <span class="highlight-o">:</span><span class="highlight-k">forall<
span> <span class="highlight-n">n</span> <span class="highlight-o">:</span> <span clas
="highlight-kt">nat</span><span class="highlight-o">,</span>
</span></span><span class="highlight-line"><span class="highlight-cl"> <span class="hi
hlight-o">( </span><span class="highlight-n">n</span> <span class="highlight-o">+ </spa
> <span class="highlight-mi">1</span><span class="highlight-o">)</span> <span class="h
hlight-o">=?</span> <span class="highlight-mi">0</span> <span class="highlight-o">=
</span> <span class="highlight-bp">>false</span><span class="highlight-o">.</span>
</span></span><span class="highlight-line"><span class="highlight-cl"><span class="high
ight-kn">Proof</span><span class="highlight-o">.</span>
</span></span><span class="highlight-line"><span class="highlight-cl"> <span class="hi
hlight-k">intros</span> <span class="highlight-n">n</span><span class="highlight-o">.</
pan> <span class="highlight-k">destruct</span> <span class="highlight-n">n</span> <sp
n class="highlight-k">as</span> <span class="highlight-o">[</span> <span class="highlig
t-n">n'</span><span class="highlight-o">]</span> <span class="highlight-n">eqn</span>
<span class="highlight-o">:</span><span class="highlight-n">E</span><span class="highl
ght-o">.</span>
</span></span><span class="highlight-line"><span class="highlight-cl"> <span class="hi
hlight-o">-</span><span class="highlight-kp">reflexivity</span><span class="highlight-o
">.</span>
</span></span><span class="highlight-line"><span class="highlight-cl"> <span class="hi
hlight-o">-</span> <span class="highlight-kp">reflexivity</span><span class="highlight-o
">.</span> <span class="highlight-kn">Qed</span><span class="highlight-o">.</span>
</span></span></code></pre>
<p>可将 <code>intros n. destruct n as [| n'] eqn:E.</code> 简写为 <code>intros [|n'].</code>
。 </p>

```